Main topics in this chapter

- **Molecular nature of matter** (atoms, molecules, Avogadro's number).
- **Ideal gas model** and basic **assumptions** of kinetic theory.
- **Kinetic interpretation of temperature**:

$$\text{average KE per molecule} = \frac{3}{2}k_{\mathrm{B}}T.$$

- **Different speeds** of gas molecules:
  - most probable speed v_p,
  - average (mean) speed $\overline{v}$,
  - r.m.s. speed v_{rms} and their relation.
- **Degrees of freedom** for monoatomic, diatomic, polyatomic gases.
- **Law of equipartition of energy** and link to **specific heats**:

$$C_V,\ C_P,\ \gamma = \frac{C_P}{C_V} \text{ for different gases.}$$

- **Mean free path** (average distance between molecular collisions).

1 Kinetic Theory of Gases: Introduction

Introduction

Kinetic theory of gases provides a microscopic picture of gases by treating them as a large collection of tiny particles (molecules) in continuous, random motion inside a container. It explains how observable quantities like pressure, temperature and internal energy emerge from the motion and collisions of these molecules.
In this chapter, the simple model of an *ideal gas* will be used: gas molecules are considered point particles that move freely, collide elastically with one another and with the container walls, and experience negligible intermolecular forces except during collisions. Using these assumptions, the macroscopic gas laws are connected to molecular motion in a quantitative way.

In this chapter you will learn

By the end of this chapter, you should be able to:

- Describe the basic assumptions of the kinetic theory model for an ideal gas.
- Explain how gas pressure arises due to molecular impacts on the walls of a container.
- Relate the temperature of a gas to the average kinetic energy of its molecules.
- Define and compare different molecular speeds: average speed, root mean square (rms) speed and most probable speed.
- Connect the idea of degrees of freedom with molar heat capacities and the ratio $\gamma = \frac{C_P}{C_V}$.
- State the main limitations of the ideal gas model and identify conditions where real gases deviate from ideal behaviour.

2 Assumptions of Kinetic Theory of Gases

The kinetic theory of gases describes an ideal gas using a simple microscopic model based on the following assumptions:

1. A gas consists of a very large number of molecules, all identical for a given gas, moving in continuous, random motion in all directions.
2. The actual volume occupied by the gas molecules is negligibly small compared to the total volume of the container. The molecules are treated as point particles.

3. There are no intermolecular forces of attraction or repulsion between molecules except during collisions. Between collisions, each molecule moves in a straight line with uniform velocity.
4. Collisions between molecules, and between molecules and the walls of the container, are perfectly elastic. The total kinetic energy of the molecules is conserved in collisions.
5. The time spent by molecules in collisions is extremely small compared to the time between successive collisions.
6. For a given sample of gas at fixed temperature, molecules have a range of speeds, but the distribution of speeds remains steady in time, and macroscopic quantities such as pressure and temperature represent averages over many molecules and collisions.

3 Mean Free Path: Definition

Microscopic Definition of Mean Free Path

Consider a single gas molecule undergoing many successive collisions. Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ be the distances travelled by the molecule between the first and second collision, between the second and third collision, and so on, up to the n-th interval.
The *mean free path* λ_m is defined as the average of these distances:

$$\lambda_m = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n}{n}.$$

4 Average Relaxation Time

Microscopic Definition of Average Relaxation Time

Consider a gas molecule undergoing many successive collisions. Let $\tau_1, \tau_2, \tau_3, \ldots, \tau_n$ be the time intervals between one collision and the next, for a large number of collisions.
The *average relaxation time*, denoted by τ, is defined as the average of these time intervals:

$$\tau = \frac{\tau_1 + \tau_2 + \tau_3 + \cdots + \tau_n}{n}.$$

5 Boyle's Law

Statement of Boyle's Law:
For a fixed mass of a gas kept at constant temperature (isothermal process), the pressure of the gas is inversely proportional to its volume. In other words, if temperature and amount of gas remain unchanged, an increase in pressure causes a proportional decrease in volume, and vice versa.
Mathematically, Boyle's law can be written as

$$P \propto \frac{1}{V}$$

or

$$PV = \text{constant}$$

for a given mass of gas at constant temperature.
If the state of the gas changes from (P_1, V_1) to (P_2, V_2) at the same temperature, then

$$P_1 V_1 = P_2 V_2.$$

6 Charles's Law

Statement of Charles's Law
For a fixed mass of a gas kept at constant pressure, the volume of the gas is **directly proportional** to its absolute temperature (measured in kelvin). In simple words, if the pressure and amount of gas remain unchanged, increasing the temperature causes the gas volume to increase in the same ratio, and decreasing the temperature causes the volume to decrease in the same ratio.
Mathematical Form of Charles's Law
If V is the volume and T is the absolute temperature (in K), then at constant pressure:

$$V \propto T \quad \Rightarrow \quad \frac{V}{T} = \text{constant}.$$

For a given mass of gas changing from an initial state (V_1, T_1) to a final state (V_2, T_2) at the same pressure, Charles's law can be written as

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

Note: The temperature must always be taken in kelvin when applying gas laws.

7 Combined Gas Law

We consider a fixed mass of gas undergoing changes in pressure, volume and temperature. From Boyle's and Charles's laws, we can obtain a single relation connecting P, V and T.

Step 1: Boyle's and Charles's Laws

- **Boyle's law (at constant temperature):**

$$PV = \text{constant} \quad \Rightarrow \quad P_1V_1 = P_2V_2.$$

- **Charles's law (at constant pressure):**

$$\frac{V}{T} = \text{constant} \quad \Rightarrow \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

Step 2: Expressing P, V and T in One Constant

For a fixed mass of gas, experimental results show:

$$P \propto \frac{1}{V} \quad (\text{at constant } T), \qquad V \propto T \quad (\text{at constant } P).$$

Combining these dependences, we get

$$P \propto \frac{T}{V} \quad \Rightarrow \quad \frac{PV}{T} = \text{constant}$$

for a given mass of gas. [web:66][web:78]

Step 3: Two States of the Same Gas

Suppose the gas changes from an initial state (P_1, V_1, T_1) to a final state (P_2, V_2, T_2). Since $\dfrac{PV}{T}$ is constant for this fixed mass, we can write

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}.$$

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

This relation is called the **combined gas law**. It is very useful when all three variables P, V and T change for a fixed amount of gas.

8 Ideal Gas (Perfect Gas) Equation

From Combined Gas Law to Ideal Gas Equation

For a fixed mass of a gas, experiments show that the quantity

$$\frac{PV}{T}$$

remains constant when the gas changes from one state to another. This gives the *combined gas law*:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

for the same mass of gas. [web:66][web:78]

Let us denote this constant for a given sample of gas by K:

$$\frac{PV}{T} = K.$$

Dependence on Amount of Gas

If we take:

- one sample with mass m_1 of gas, and
- another sample with mass m_2 of the *same gas*,

then experiments show that the constant K is **directly proportional** to the amount of gas present. In other words, if we double the amount of gas (keeping the same gas and same conditions), $\frac{PV}{T}$ also doubles.

This means:

$$K \propto n,$$

where n is the *number of moles* of gas. Therefore we can write

$$K = nR,$$

where R is a universal constant called the **universal gas constant**. [web:66][web:78]

Substituting $K = nR$ into $\frac{PV}{T} = K$, we get

$$\frac{PV}{T} = nR.$$

Rearranging,

$$PV = nRT.$$

Ideal Gas (Perfect Gas) Equation:

$$\boxed{PV = nRT}$$

where

- P is the pressure of the gas,
- V is the volume of the gas,
- T is the absolute temperature (in kelvin),
- n is the number of moles of the gas,
- R is the universal gas constant.

Useful Forms and Values

If the gas has mass m and molar mass (molecular weight) M, then

$$n = \frac{m}{M},$$

so the ideal gas equation can also be written as

$$PV = \frac{m}{M}RT.$$

The value of the universal gas constant R is

$$R = 8.314 \text{ J mol}^{-1}\text{K}^{-1}$$

in SI units.

Physical Meaning for Students

- The equation $PV = nRT$ tells us that $\frac{PV}{T}$ is the same for *one mole of any ideal gas*; this is why R is called a universal constant.
- All simple gas laws are contained inside this one equation:
  - If n and T are constant, $PV =$ constant (Boyle's law).
  - If n and P are constant, $\frac{V}{T} =$ constant (Charles's law).
  - If n and V are constant, $\frac{P}{T} =$ constant (Gay-Lussac's law).
- $PV = nRT$ is the starting point for almost every numerical involving gases: changing state, mixing gases, or connecting microscopic kinetic theory to macroscopic variables.

How Do We Find the Value of R?

The ideal gas equation is

$$PV = nRT.$$

To find the value of the universal gas constant R, we choose conditions where P, V, n and T are known from experiments, and then solve for R.

Using One Mole of Gas at STP

Consider **one mole** of an ideal gas at standard temperature and pressure (STP).

- Amount of gas: $n = 1$ mol.
- Standard temperature: $T = 273$ K (i.e. 0°C).
- Standard pressure: $P = 1$ atm $= 1.013 \times 10^5$ Pa.
- Experimental result: One mole of an ideal gas at STP occupies approximately

$$V = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3.$$

Substitute these values into $PV = nRT$:

$$PV = nRT \quad \Rightarrow \quad R = \frac{PV}{nT}.$$

So

$$R = \frac{(1.013 \times 10^5 \text{ Pa}) \times (22.4 \times 10^{-3} \text{ m}^3)}{(1 \text{ mol}) \times (273 \text{ K})}.$$

Numerical Value

Calculate the numerator:

$$PV \approx 1.013 \times 10^5 \times 22.4 \times 10^{-3} \approx 1.013 \times 22.4 \times 10^2 \approx 22.7 \times 10^2 \approx 2.27 \times 10^3 \text{ J}.$$

Now divide by $T = 273$ K:

$$R \approx \frac{2.27 \times 10^3 \text{ J}}{273 \text{ K}} \approx 8.31 \text{ J mol}^{-1}\text{K}^{-1}.$$

Universal Gas Constant:

$$\boxed{R \approx 8.31 \text{ J mol}^{-1}\text{K}^{-1}}$$

This same value of R works for *any* ideal gas because one mole of all ideal gases obeys $PV = nRT$ in the same way.

What Students Should Remember

- R is not "derived" from theory first; it is **measured** using experimental values of P, V, n and T and then used as a universal constant.
- For PUC/CET, remember:
$$R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$$
and the idea that
$$R = \frac{PV}{nT}$$
for any state of an ideal gas.

9 Boltzmann Constant k_B

Definition

In kinetic theory, we often use the ideal gas equation in two forms:

$$PV = nRT \quad \text{(moles form)}, \qquad PV = Nk_BT \quad \text{(molecules form)}.$$

Here:

- n = number of moles,
- N = number of molecules,
- R = universal gas constant,
- k_B = Boltzmann constant.

Relation between R and k_B

Number of molecules and number of moles are related by

$$N = nN_A,$$

where N_A is Avogadro's number.

Equating the two forms of the gas equation:

$$PV = nRT = Nk_BT = nN_Ak_BT.$$

Cancel nT from both sides:

$$R = N_Ak_B.$$

Therefore,

$$\boxed{k_B = \frac{R}{N_A}}.$$

Using

$$R \approx 8.31 \text{ J mol}^{-1}\text{K}^{-1}, \qquad N_A \approx 6.022 \times 10^{23} \text{ mol}^{-1},$$

we get

$$k_B \approx \frac{8.31}{6.022 \times 10^{23}} \approx 1.38 \times 10^{-23} \text{ J K}^{-1}.$$

Boltzmann Constant:

$$\boxed{k_B \approx 1.38 \times 10^{-23} \text{ J K}^{-1}}$$

It gives the energy scale per *single molecule* per kelvin and is used in kinetic theory formulas like

$$\bar{K} = \frac{3}{2}k_BT$$

for the average kinetic energy of one gas molecule.

When to Use R and When to Use k_B

Moles $\Rightarrow R$, Molecules $\Rightarrow k_B$

Two Forms of the Ideal Gas Equation

The ideal gas equation can be written in two equivalent ways:

$$PV = nRT \quad \text{and} \quad PV = Nk_BT.$$

- n: number of moles of gas.
- N: number of molecules (or particles) in the gas.
- R: universal gas constant.
- k_B: Boltzmann constant.

They are related by

$$R = N_A k_B,$$

where N_A is Avogadro's number.

Simple Rule for Students

Which one to use?

- Use **R** when the question talks about **moles** or uses the symbol n. Example: "2 moles of an ideal gas are kept at temperature 300 K..."
$$PV = nRT.$$
- Use $\mathbf{k_B}$ when the question talks about **single molecules** or uses the symbol N. Example: "Find the average kinetic energy of one molecule at 300 K..."
$$\bar{K} = \frac{3}{2}k_BT.$$

10 Kinetic Gas Equation (Simple Derivation)

We want a simple relationship between the **pressure** of a gas and the **motion of its molecules**. The kinetic gas equation connects pressure with the **mass**, **number of molecules** and the **average of the square of their speed**.

Model and assumptions (very simple)

- Consider a gas in a container of volume V.
- There are N identical molecules, each of mass m.
- Molecules move randomly and collide elastically with the walls.
- For simplicity, assume that the **average speed** of the molecules is v, and that this speed does not change during collisions.

Idea of pressure from molecular impacts

Pressure is **force per unit area**. The gas exerts force on the walls by **molecules hitting the walls and bouncing back**.

The faster and heavier the molecules, and the more often they hit the walls, the **greater** is the pressure.

We use two facts:

- When a molecule hits a wall and bounces back, it changes its momentum. This change in momentum gives a **force** on the wall.
- The force from many molecules together gives the **pressure**.

Average effect of all molecules

Without going into full vector detail, the result (from more detailed calculation) can be written in a simple form:

$$PV = \frac{1}{3}Nm\,\overline{v^2},$$

where

- P is the pressure of the gas,
- V is the volume of the gas,
- N is the number of molecules,
- m is the mass of each molecule,
- $\overline{v^2}$ is the **mean square speed** of the molecules (average of v^2 over all molecules).

Kinetic gas equation (simple form)

$$\boxed{PV = \frac{1}{3}Nm\,\overline{v^2}}$$

If we define the **mass of gas** as $M = Nm$, we can also write:

$$P = \frac{1}{3}\frac{M}{V}\,\overline{v^2},$$

and since $\rho = \dfrac{M}{V}$ is the density of the gas,

$$\boxed{P = \frac{1}{3}\rho\,\overline{v^2}}.$$

Meaning of the result

- If molecules move **faster** ($\overline{v^2}$ increases), pressure P increases.
- If there is **more mass per unit volume** (greater ρ), pressure P also increases.

So, pressure of a gas is directly related to how **fast** and how **massive** its molecules are, and how many of them are present in a given volume.

11 Root Mean Square (r.m.s.) Velocity

Gas molecules in a container do not all have the **same speed**. Some move slowly, some very fast, and most have speeds in between. To describe this spread of speeds in a simple way, we define the **root mean square (r.m.s.) velocity**.

11.1 Definition

Consider a gas containing a large number of molecules with speeds

$$v_1, v_2, v_3, \ldots, v_N.$$

Root mean square (r.m.s.) velocity v_{rms} is defined as:

- Take the **square** of each speed: $v_1^2, v_2^2, \ldots, v_N^2$.
- Find the **mean** (average) of these squares:

$$\overline{v^2} = \frac{v_1^2 + v_2^2 + \cdots + v_N^2}{N}.$$

- Take the **square root** of this mean:

$$v_{\text{rms}} = \sqrt{\overline{v^2}}.$$

So,

$$\boxed{v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_N^2}{N}}}.$$

11.2 Physical meaning

- The r.m.s. velocity is a kind of **average speed**, but it gives **more weight** to higher speeds because of the squaring.
- It is directly related to the **average kinetic energy** of gas molecules. If temperature increases, v_{rms} increases.

From kinetic theory and the ideal gas equation, we can also write r.m.s. speed in terms of **temperature** and **molar mass**:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where

- R is the universal gas constant,
- T is the absolute temperature (in kelvin),
- M is the molar mass of the gas (in kg mol^{-1}).

Thus, $v_{\text{rms}} \propto \sqrt{T}$ and $v_{\text{rms}} \propto \dfrac{1}{\sqrt{M}}$: at the same temperature, **lighter gases** have **higher** r.m.s. speeds.

Derivation of $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

We will combine:

- the **kinetic gas equation** from kinetic theory, and
- the **ideal gas equation** from thermodynamics,

to obtain a formula for the **root mean square speed** of gas molecules.

Step 1: Kinetic theory relation

From kinetic theory, the pressure of an ideal gas is

$$P = \frac{1}{3}\,\rho\,\overline{v^2},$$

where

- ρ is the mass density of the gas,
- $\overline{v^2}$ is the mean of the square of molecular speeds.

Define

$$v_{\text{rms}} = \sqrt{\overline{v^2}} \quad \Rightarrow \quad \overline{v^2} = v_{\text{rms}}^2.$$

So,

$$P = \frac{1}{3}\,\rho\, v_{\text{rms}}^2.$$

Step 2: Express density in terms of mass and volume

Let

- M be the **mass of one mole** of the gas (molar mass, in kg mol^{-1}),
- n be the number of moles,
- total mass of gas $= nM$,
- volume of the gas $= V$.

Then mass density

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{nM}{V}.$$

Substitute this in the kinetic theory expression:

$$P = \frac{1}{3} \cdot \frac{nM}{V}\, v_{\text{rms}}^2.$$

Step 3: Use the ideal gas equation

For an ideal gas,

$$PV = nRT,$$

where R is the universal gas constant and T is the absolute temperature.

From the kinetic expression, multiply both sides by V:

$$PV = \frac{1}{3} nM v_{\text{rms}}^2.$$

But from the ideal gas equation, the same PV is also equal to nRT. Therefore,

$$nRT = \frac{1}{3} nM v_{\text{rms}}^2.$$

Cancel n (assuming $n \neq 0$):

$$RT = \frac{1}{3} M v_{\text{rms}}^2.$$

Step 4: Solve for v_{rms}

Rearrange:

$$v_{\text{rms}}^2 = \frac{3RT}{M}.$$

Taking square root on both sides,

$$\boxed{v_{\text{rms}} = \sqrt{\frac{3RT}{M}}}.$$

Key points from the formula

- $v_{\text{rms}} \propto \sqrt{T}$: higher temperature $\Rightarrow$ higher r.m.s. speed.
- $v_{\text{rms}} \propto \dfrac{1}{\sqrt{M}}$: at the same T, lighter gases (smaller M) have higher r.m.s. speeds than heavier gases.

Degrees of Freedom

The **degrees of freedom** of a molecule are the number of **independent ways** in which it can **move** (or store energy) — by **translation**, **rotation** or **vibration**.
Each independent mode of motion is one **degree of freedom**.

Types of degrees of freedom

- **Translational**: motion of the whole molecule from one place to another along x, y, z axes (up–down, left–right, forward–back). A free molecule in space has **3 translational** degrees of freedom.
- **Rotational**: rotation of the molecule about various axes passing through its centre of mass. Number of rotational degrees depends on the shape of the molecule.
- **Vibrational**: periodic stretching and compressing of bonds (atoms oscillating relative to each other); important mainly at high temperatures for diatomic/polyatomic gases.

Common cases for gases

Type of gas	Translational	Rotational	Total f (at ordinary T)
Monoatomic (He, Ne, Ar)	3	0	3
Diatomic (H_2, N_2, O_2)	3	2	5
Linear triatomic (CO_2)	3	2	5
Nonlinear triatomic (H_2O)	3	3	6

Why monoatomic gas has 3 degrees of freedom?
A monoatomic molecule is just a point mass. It can move in three perpendicular directions (x, y, z), so it has **3 translational** degrees and practically no rotational structure to consider.

Why diatomic gas has 5 degrees of freedom (at normal T)?
A diatomic molecule can:

- translate along $x, y, z \Rightarrow 3$ translational,
- rotate about two perpendicular axes through its centre of mass (rotation about its own axis is negligible) $\Rightarrow 2$ rotational.

So total $f = 3 + 2 = 5$ degrees of freedom.

These degrees of freedom are used in the next part (equipartition of energy) to calculate average kinetic energy per molecule and specific heats of gases.

12 Law of Equipartition of Energy

Gas molecules can move in different independent ways (degrees of freedom). The **law of equipartition of energy** tells us how the total energy of a gas is **shared equally** among these degrees of freedom when the gas is in **thermal equilibrium** at temperature T.

12.1 Energy of a single molecule

Consider a molecule of mass m moving with velocity components v_x, v_y, v_z along the three mutually perpendicular axes x, y, z. The **kinetic energy** of this single molecule is

$$\varepsilon_t = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2.$$

For a gas in **thermal equilibrium** at temperature T, we consider the **average** kinetic energy $\langle\varepsilon_t\rangle$ per molecule:

$$\langle\varepsilon_t\rangle = \left\langle\frac{1}{2}mv_x^2\right\rangle + \left\langle\frac{1}{2}mv_y^2\right\rangle + \left\langle\frac{1}{2}mv_z^2\right\rangle.$$

Because there is **no preferred direction** (the gas is isotropic), the average energies in the three directions are equal:

$$\left\langle \frac{1}{2}mv_x^2 \right\rangle = \left\langle \frac{1}{2}mv_y^2 \right\rangle = \left\langle \frac{1}{2}mv_z^2 \right\rangle.$$

From kinetic theory, the total average translational kinetic energy per molecule is

$$\langle \varepsilon_t \rangle = \frac{3}{2}k_B T,$$

where k_B is the Boltzmann constant.
Hence each of the three quadratic terms contributes:

$$\left\langle \frac{1}{2}mv_x^2 \right\rangle = \left\langle \frac{1}{2}mv_y^2 \right\rangle = \left\langle \frac{1}{2}mv_z^2 \right\rangle = \frac{1}{2}k_B T.$$

Law of Equipartition of Energy (statement)

For a system in **thermal equilibrium** at temperature T, the **average energy** associated with each **quadratic degree of freedom** is

$$\boxed{\frac{1}{2}k_B T \text{ per molecule per degree of freedom.}}$$

If a molecule has f degrees of freedom, its **average energy** is

$$\boxed{\langle \varepsilon \rangle = \frac{f}{2}k_B T.}$$

12.2 Internal energy of an ideal gas

For N molecules, each with f degrees of freedom, the **total internal energy** (microscopic kinetic energy) is

$$U = N\langle \varepsilon \rangle = N \cdot \frac{f}{2}k_B T.$$

If there are n moles of gas, then $N = nN_A$ and $k_B N_A = R$ (gas constant). So:

$$U = n \cdot \frac{f}{2}RT.$$

Average energy per mole and per molecule

- Per molecule: $\langle \varepsilon \rangle = \frac{f}{2}k_B T.$
- Per mole: $U_{\text{per mole}} = \frac{f}{2}RT.$

12.3 Applications to monoatomic and diatomic gases

Gas	f	Average energy per molecule	Molar internal energy
Monoatomic (He, Ne, Ar)	3	$\frac{3}{2}k_B T$	$U = \frac{3}{2}nRT$
Diatomic (at ordinary T)	5	$\frac{5}{2}k_B T$	$U = \frac{5}{2}nRT$

(For diatomic gases at ordinary temperatures, 3 translational + 2 rotational degrees of freedom are active; vibrational modes become important only at high temperatures.)

12.4 Connection with specific heats (brief)

At constant volume,

$$C_V = \left(\frac{1}{n}\frac{dU}{dT}\right)_V = \frac{f}{2}R.$$

So:

- For monoatomic gas ($f = 3$):

$$C_V = \frac{3}{2}R, \quad C_P = C_V + R = \frac{5}{2}R, \quad \gamma = \frac{C_P}{C_V} = \frac{5}{3}.$$

- For diatomic gas ($f = 5$):

$$C_V = \frac{5}{2}R, \quad C_P = \frac{7}{2}R, \quad \gamma = \frac{C_P}{C_V} = \frac{7}{5}.$$

What should students remember?

- Law: each quadratic degree of freedom has $\frac{1}{2}k_BT$ energy per molecule.
- For a molecule with f degrees of freedom:

$$\langle \varepsilon \rangle = \frac{f}{2}k_BT, \quad U = \frac{f}{2}nRT.$$

- Monoatomic gas: $f = 3,\ C_V = \frac{3}{2}R,\ C_P = \frac{5}{2}R,\ \gamma = \frac{5}{3}$.
- Diatomic gas (normal T): $f = 5,\ C_V = \frac{5}{2}R,\ C_P = \frac{7}{2}R,\ \gamma = \frac{7}{5}$.